Calculation of f_B and the "Isgur-Wise Function" using a non-perturbatively improved fermion action

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We present a calculation of f_B and of the form factors for the semi-leptonic decay $\overline{B} \to Dl\bar{\nu}$ in the quenched approximation to QCD. Results are generated on lattices at $\beta = 6.2$, using an $O(a)$-improved fermion action, with the clover coefficient determined non-perturbatively.

1. Introduction

A knowledge of the hadronic matrix elements for the decays of mesons containing heavy quarks is essential to the experimental determination of elements of the CKM matrix involving the heavy quarks. In this contribution, we concentrate on the hadronic matrix elements for the semi-leptonic decay $\bar{B} \to Dl\bar{\nu}$, and also discuss f_B. The calculations of the $D \to K$ and $D \to \pi$ matrix elements are contained in the talk of Chris Maynard[1]. An important element of this programme is the use of a "clover" fermion action with the clover coefficient, $c_{\rm SW}$, determined non-perturbatively. Thus we expect a substantial reduction in discretisation errors compared to earlier calculations using the tree-level value of $c_{\rm SW}$.

2. Simulation details

The calculation is performed in the quenched approximation on an ensemble of 216 $24^3 \times 48$ lattices at $\beta = 6.2$, generated using the Wilson gauge action. In the calculation of the the semi-leptonic decay matrix element, we used four values of the final heavy-quark mass, corresponding to $\kappa_h = 0.1200, 0.1233, 0.1266$ and 0.1299, and two values of the initial heavy-quark mass, corresponding to $\kappa_h = 0.1200$ and 0.1266; the charm-quark mass corresponds roughly to $\kappa = 0.1233$. Further details of the calculation are contained in ref. [1].

In the non-perturbatively improved scheme, the axial vector and pseudoscalar currents mix:

$$\begin{aligned} A_\mu^{\rm R} &= Z_A\big(1 + b_A a m_q\big) \\ &\quad \times \Big[A_\mu^{\rm latt} + c_A \frac{a}{2}(\partial_\mu^* + \partial_\mu)P^{\rm latt}\Big] \end{aligned} \quad (1)$$

where

$$m_q = \frac{1}{2}\Big(\frac{1}{\kappa} - \frac{1}{\kappa_{\rm crit}}\Big). \quad (2)$$

We take the ALPHA Collaboration's numerical values[2] for the overall renormalisation factor, Z_A and for c_A, and the one-loop calculation of b_A. Note that this form is modified for the case of a current constructed of non-degenerate quarks through the addition of a term linear in the difference of the quark masses, with coefficient b'_A; this coefficient not been computed.

The renormalisation of the vector current proceeds likewise:

$$\begin{aligned} V_\mu^{\rm R} &= Z_V(1 + b_V a m_q) \times \\ &\quad \Big[V_\mu^{\rm latt} + c_V \frac{a}{2}(\partial_\nu^* + \partial_\nu)\Sigma_{\mu\nu}^{\rm latt}\Big], \end{aligned} \quad (3)$$

where Z_V, b_V and c_V are all known non-perturbatively.

3. Pseudoscalar decay constant

We obtain f_P/f_π at each value of κ_l and κ_h, where P is the heavy-light pseudoscalar, and f_π is the decay constant for a light meson constructed from degenerate quarks of mass corresponding to κ_l. We then extrapolate κ_l to $\kappa_{\rm crit}$ at fixed κ_h. Note that, whilst Z_A cancels between the denominator and numerator, an effective relative renormalisation enters through b_A.

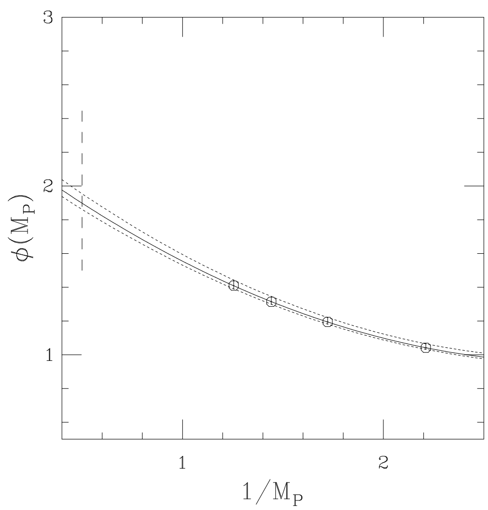

Figure 1. The ratio $\phi(M_p)$ is shown at $\kappa_l = \kappa_{\rm crit}$ for each value of κ_h. The solid line is a quadratic fit to the data, and the dashed line corresponds to the extrapolation to M_B.

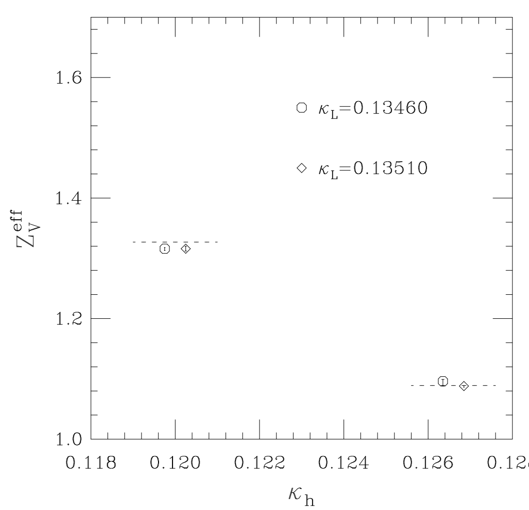

Figure 2. $Z_V^{\rm eff}$ is shown for two values of the light-quark mass and for two values of the heavy-quark mass. The dashed lines represent the predictions using the non-perturbative values of eqn. 3

To investigate the heavy-mass dependence, we construct the scaling quantity

$$\Phi \equiv (\alpha_s(M_P)/\alpha_s(M_B))^{2/\beta_0}(f_P/f_\pi)\sqrt{M_P}, \qquad (4)$$

where we take $N_f = 0$, and perform a quadratic fit in $1/M_P$, as shown in Figure 1. Setting $a^{-1} = 2.64\,\mathrm{GeV}$, the value obtained from m_ρ, we find

$$\begin{aligned} f_D/f_\pi &= 1.19 \ {}^{+3}_{-1} \\ f_B/f_\pi &= 1.34 \ {}^{+4}_{-3}, \end{aligned} \qquad (5)$$

where the quoted errors are purely statistical, and hence

$$\begin{aligned} f_D &= 190 \ {}^{+5}_{-2} \ \mathrm{MeV} \\ f_B &= 176 \ {}^{+5}_{-4} \ \mathrm{MeV}. \end{aligned} \qquad (6)$$

4. Semi-leptonic decays and the Isgur-Wise function

The form factors for the semi-leptonic decay $\overline{P} \to P' l \bar{\nu}$, where $P^{(')}$ is a heavy-light pseudoscalar meson, may be parametrised as

$$\frac{\langle P'(\mathbf{p}')|\bar{Q}'\gamma_\mu Q|P(\mathbf{p})\rangle}{\sqrt{M_P M_{P'}}} = (v+v')_\mu h^+(\omega; m_Q, m_{Q'}) + (v-v')_\mu h^-(\omega; m_Q, m_{Q'}). \qquad (7)$$

Here $v^{(')}$ is the four velocity of the initial (final) meson, and $\omega = v \cdot v'$. In the Heavy Quark Effective Theory (HQET), the form factors display an additional spin-flavour symmetry, and are related to a universal "Isgur-Wise" function $\xi(\omega)$:

$$h^i(\omega; m_Q, m_{Q'}) = \xi(\omega) \times (\alpha^i + \beta^i(\omega; m_Q, m_{Q'}) + \gamma^i(\omega; m_Q, m_{Q'})) \qquad (8)$$

where $\alpha^+ = 1$, $\alpha^- = 0$, and β^i and γ^i represent the radiative and power corrections respectively. Note that ξ is normalised: $\xi(\omega = 1) = 1$.

For the case of degenerate transitions at zero momentum transfer, we can obtain a direct measurement of the effective renormalisation constant

$$Z_V^{\rm eff} = Z_V(1 + b_V a m_Q), \qquad (9)$$

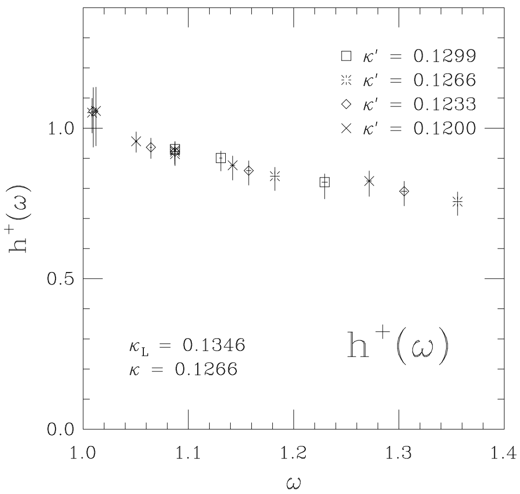

Figure 3. $h^+(\omega)$ is shown for fixed light-quark and initial heavy-quark masses.

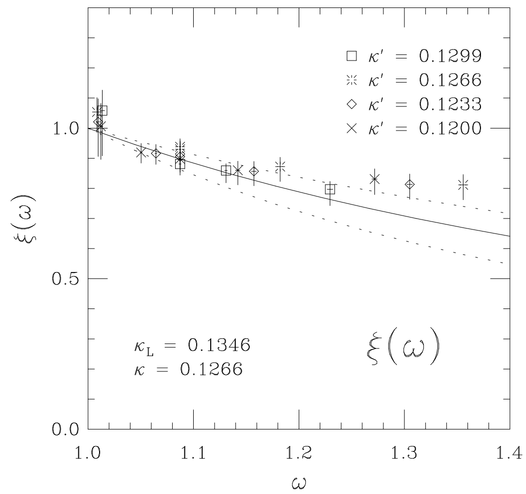

Figure 4. $\xi(\omega) \equiv h^+(\omega)/(1+\beta^+(\omega))$ is shown for the same parameter values as in Figure 3. The solid line is a fit to the data for $\kappa' = 0.1233$, as described in the text.

and compare with the value using the non-perturbative parameters of eqn. 3, as shown in Figure 2. The agreement is striking, and perhaps surprising since the NP prescription only removes $\mathcal{O}(am_Q)$ errors.

In Figure 3, we show the form factor $h^+(\omega)$ for fixed value of the light-quark mass, close to the strange, for fixed initial heavy-quark mass, and for four values of the final heavy-quark mass; as expected, h^- is very small. We compute β^+using Neubert's short-distance expansion of the currents[3], and "define" the Isgur-Wise function through

$$\xi(\omega) = \frac{h^+(\omega)}{1+\beta^+(\omega)}. \tag{10}$$

The corrected form factor is shown in Figure 4, together with a one-parameter fit for $\omega < 1.2$ to

$$\xi(\omega) = \frac{2}{1+\omega}\exp(-(2\rho^2-1)(\omega-1)/(\omega+1)).$$

The use of the NP prescription has enabled a far more satisfactory treatment of discretisation errors than the calculation using the tree-level-improved SW action[4]. It remains to obtain a quantitative estimate of the power corrections, and a determination of the remaining form factors.

5. Acknowledgements

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